SE CONFIDENCE INTERVALS

STEP-BY-STEP EXAMPLES

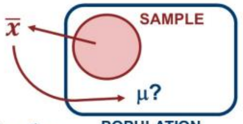
Let's do some examples

Review: confidence intervals

Watch our intro to confidence intervals video for more about this

We want to know the population mean (μ).
We can't measure the whole population.
We take a random sample.
We calculate a sample mean ($\bar{x}$).

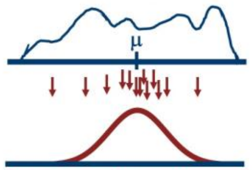
The sample mean is an estimate of the population mean, but sampling error makes it inexact.



We specify a region which probably includes the population mean, based on the sample mean.

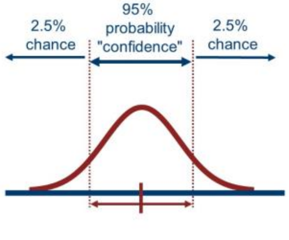
Review: confidence intervals
The Central Limit Theorem

1. Distribution of sample means from a population is normally distributed.
2. Any given sample is probably from middle region of this distribution.
3. We don't know the population variance, so we estimate it from the sample.
4. To avoid bias and underestimating the population variance we use a t-distribution to describe middle region.



Technically you can make confidence intervals with a normal distribution, but you shouldn't

Review: confidence intervals
The t-distribution



Since sample is probably inside the t-distribution for the samples, we create a t-distribution around our sample mean, the center of that *probably* corresponds to a region around the population mean.

We just need to identify that middle region.

Example #1 - 16 values

$SE = 1.25$

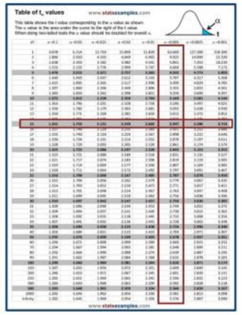
What is the **99%** confidence interval for the population mean?

Degrees of freedom (df) is n-1.
Since n=16, **df = 15**.

99% confidence interval uses 0.5%, **α=0.005** on each side.

t-value to use is therefore **2.947**

99% CI: 12 ± (2.947)(1.25)
99% CI: 12 ± 3.68375, {8.316 , 15.684}



Example #1 - 16 values

$SE = 1.25$

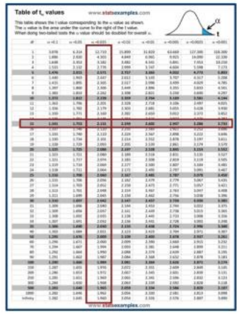
What is the **95%** confidence interval for the population mean?

Degrees of freedom (df) is n-1.
Since n=16, **df = 15**.

95% confidence interval uses 2.5%, **α=0.025** on each side.

t-value to use is therefore **2.131**

95% CI: 12 ± (2.131)(1.25)
95% CI: 12 ± 2.66375, {9.336,14.664}



Example #1 - 16 values

Consider a sample of 16 values from a population with a mean of 12 and standard deviation of 5.0.

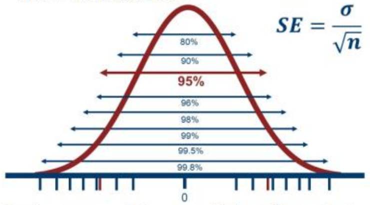
What is the 95% confidence interval for the population mean?
What is the 99% confidence interval for the population mean?
What is the 99.8% confidence interval for the population mean?

First step, calculate the standard error:

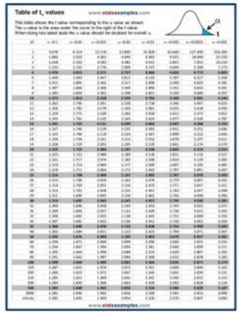
$$SE = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{5.0}{\sqrt{16}} = \frac{5.0}{4} = 1.25$$

Review: confidence intervals
The t-distributions

$$SE = \frac{\sigma}{\sqrt{n}}$$



Regions around $\bar{x}$ are multiples of standard errors (SE), varies by degree of freedom



Example #1 - 16 values

$SE = 1.25$

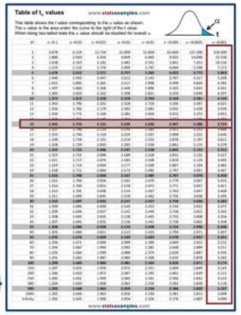
What is the **99.8%** confidence interval for the population mean?

Degrees of freedom (df) is n-1.
Since n=16, **df = 15**.

99.8% confidence interval uses 0.1%, **α=0.001** on each side.

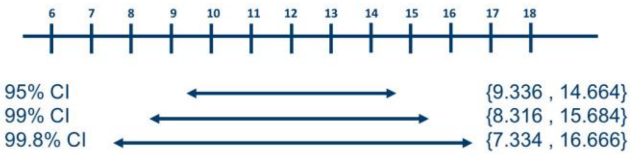
t-value to use is therefore **3.733**

99.8% CI: 12 ± (3.733)(1.25)
99.8% CI: 12 ± 4.66625, {7.334 , 16.666}



Example #1 - 16 values

n=16, $\bar{x}$=12, s=5.0



Tradeoff between degree of confidence and width of interval

Example #2 - 95% for varied sample sizes

Consider a sample of values from a population with a mean of 18 and standard deviation of 5.2.

What is the 95% CI for n = 9?
What is the 95% CI for n = 18?
What is the 95% CI for n = 36?

Unlike the previous example, we can't calculate a single standard error since the sample sizes differ.

Example #2 - 95% for varied sample sizes

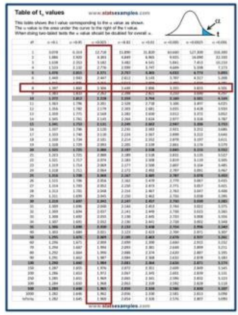
$\bar{x} = 18, s = 5.2$

What is the 95% CI for n = 9?

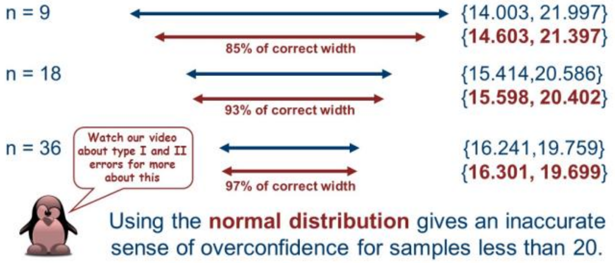
95% CI uses 2.5%, **α=0.025** on each side.
t-value to use is therefore **2.306**
df = n-1 so **df** = 9 - 1 = **8**.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{5.2}{\sqrt{9}} = \frac{5.2}{3} = 1.7333$$

95% CI: 18 ± (2.306)(1.7333)
95% CI: 18 ± 3.99699, {14.003, 21.997}



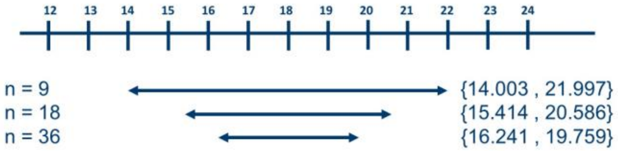
What if we use the normal dist. instead of t-distribution?



Using the **normal distribution** gives an inaccurate sense of overconfidence for samples less than 20.

Example #1 - 16 values

$\bar{x}$ = 18, s = 5.2, 95% CI



Diminishing returns for increased sample size.
x2/x4 sample size reduced CI to 65% and 46% width

Example #2 - 95% for varied sample sizes

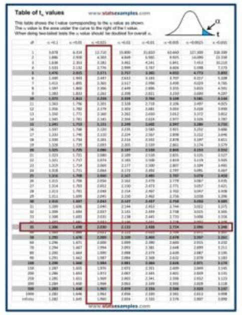
$\bar{x} = 18, s = 5.2$

What is the 95% CI for n = 36?

95% CI uses 2.5%, **α=0.025** on each side.
df = n-1 so **df** = 36 - 1 = **35**.
t-value to use is therefore **2.030**

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{5.2}{\sqrt{36}} = \frac{5.2}{6} = 0.8667$$

95% CI: 18 ± (2.030)(0.8667)
95% CI: 18 ± 1.759, {16.241 , 19.759}



Example #2 - 95% for varied sample sizes

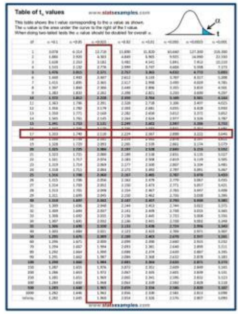
$\bar{x} = 18, s = 5.2$

What is the 95% CI for n = 18?

95% CI uses 2.5%, **α=0.025** on each side.
df = n-1 so **df** = 18 - 1 = **17**.
t-value to use is therefore **2.110**

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{5.2}{\sqrt{18}} = \frac{5.2}{4.2426} = 1.2257$$

95% CI: 18 ± (2.110)(1.2257)
95% CI: 18 ± 2.586, {15.414 , 20.586}



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